

A NEW ALGORITHM FOR OBTAINING THE RELATION BETWEEN A POINT AND
A CONTOUR DEFINED BY PRIMITIVES

Dora Florea*

*Department of Computer, Polytechnical University of Timisoara, Faculty of Automatic and Computer, Bul.Parvan nr.2, Timisoara,Romania

ABSTRACT

This paper propose an original algorithm which establish the relation between a point and a contour composed from primitives: segment of straight line, arc of circle, arc of ellipse and another primitives defined by functions. Determination of relation with a contour C of a point P Relpoint $\{P,C\} \rightarrow \{INTERIOR, EXTERIOR, BELONG TO VERTEX, BELONG TO CONTOUR\}$ it make with add the predicative formulas and the algorithm it present used to logical draft.

KEYWORDS: contour from primitives,relation of a point,predicatives formulas

INTRODUCTION

Problem of position relation a point P face to a contour C in this paper is studied consider the point P and contour C in the same plan , so the position relation possible is: Relpoint $\{P, C\} \rightarrow \{INTERIOR, EXTERIOR, BELONG TO VERTEX, BELONG TO CONTOUR\}$.

The knowledge the relation of a point P face to a contour C it ask of many problem of computational geometry, artificial intelligence or another [1] [2].

It make the observation that Winston [3] propose an algorithm which is based on observation that the number of intersections between a half line which origin in the point $P(x_p, y_p)$ and the contour C ,is odd if the point is interior end it's a stake if the point is exterior. But it possible to apper the situations which are very difficult to resolve with algorithm [3].

Im this paper Dora Florea proposed an original algorithm for decided the position relation of a point face to a contour C.

THEORETICAL CONSIDERATION

The contour C it consider composed by a chain of primitives $ch_1(V_h)_{h=1..m}$, what are defined by vertexes and ther implicite functions (1).

$$ch(V_h)=\{(x_{1h},y_{1h}),(x_{2h},y_{2h}),f_h(x,y)\}_{h=1..m} \tag{1}$$

In Fig.1 it presents a contour C composed from primitives $V_{h,h=1..5}$ and possible position of a point so: P_1 interior C , P_2 exterior C , P_3 belong to vertex C , P_4 belong to contour C.

For obtaining the relation of a point with the contour C of INTERIOR and EXTERIOR , the algorithm propose computing the modul algebraic of angles sum θ_k determined by vector which have the origin in the point $P(x_p, y_p)$ and the extremities obtaining by intersection of the straight line $x=x_p$ and $y=y_p$ with the contour (fig.2).

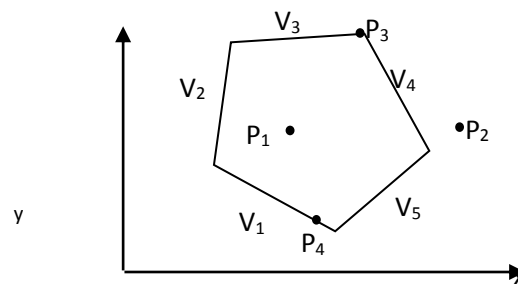
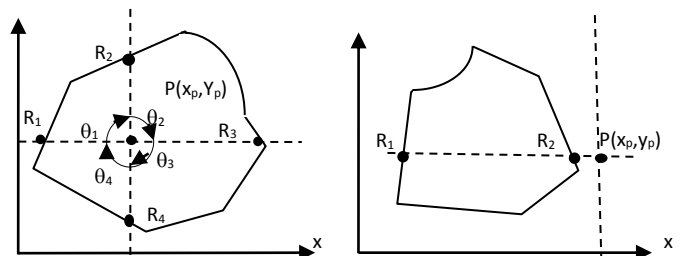


Fig. 1 Possible position of point face to contour C



a)Point $P(x_p,y_p)$ INTERIOR b)Point $P(x_p,y_p)$ EXTERIOR

Fig.2 Two position of a point $P(x_p,y_p)$

In the Fig.2(a) it presented a contour composed by 7 primitives which have a point $P(x_p, y_p)$ in the interior of contour and for the determined that the point is interior, by computer, it draw the straight line $x=x_p$ and $y=y_p$. It observed that the intersection with the contour is R_1, R_2, R_3 , and R_4 . The angles forms by the intersection R_1, R_2, R_3 , and R_4 with the point $P(x_p, y_p)$ are $\theta_1, \theta_2, \theta_3$, and θ_4 and the sum of angles is 360° .

In the Fig.2(b) it presented a contour composed by 6 primitives which have a point $P(x_p, y_p)$ in the exterior of contour and for the determined that the point is exterior, by computer, it draw the straight line $x=x_p$ and $y=y_p$. It observed that the intersection with the contour is R_1, R_2 . The angles forms by the intersection R_1, R_2 with the point $P(x_p, y_p)$ is zero.

For the computing the angle θ_k between two vectors R_kP and $R_{k+1}P$ with P common point [7], it used the relation (2) :

$$\theta_k = \frac{\text{sgn}[(\overline{V_{R_kP}} \times \overline{V_{R_{k+1}P}}) \cdot \bar{n}] \arccos[\overline{V_{R_kP}} \cdot \overline{V_{R_{k+1}P}} / (|\overline{V_{R_kP}}| |\overline{V_{R_{k+1}P}}|)]}{|\overline{V_{R_kP}}| |\overline{V_{R_{k+1}P}}|} \quad (2)$$

where \bar{n} is the unit normal vector $\bar{n} = \bar{r} \times \bar{j}$ and $n = \text{vers } \bar{P}_v \perp (\overline{V_{R_kP}}, \overline{V_{R_{k+1}P}})$.

It precises that in the relation (2) for computing the angle θ_k , it used the vectorial produce and scalar produce of two vectors $\overline{V_{R_kP}}, \overline{V_{R_{k+1}P}}$ what have the origine in the point $P(x_p, y_p)$ and the extremities in the points R_k, R_{k+1} what represents the intersections of contour C with straight lines $x=x_p, y=y_p$.

If note the vector $\overline{V_{R_kP}} = (v_{1k}, v_{2k})$ and $\overline{V_{R_{k+1}P}} = (v_{1k+1}, v_{2k+1})$, [7], where $v_{1k} = |x_k - x|, v_{2k} = |y_k - y|, v_{1k+1} = |x_{k+1} - x|, v_{2k+1} = |y_{k+1} - y|$, the relation (2) become:

$$\theta_k = \frac{\text{sgn}(v_{1k} v_{2k+1} - v_{2k} v_{1k+1}) \times \arccos\left(\frac{v_{1k} v_{1k+1} + v_{2k} v_{2k+1}}{(\sqrt{v_{1k}^2 + v_{2k}^2}) (\sqrt{v_{1k+1}^2 + v_{2k+1}^2})}\right)}{(\sqrt{v_{1k}^2 + v_{2k}^2}) (\sqrt{v_{1k+1}^2 + v_{2k+1}^2})} \quad (3)$$

In the relation (2) the angle θ_k has the positive sign if the trihedral angle $(\overline{V_{R_kP}}, \overline{V_{R_{k+1}P}}, \bar{P}_v)$ is direct.

Author Dora Florea in this paper propose the interference rules which lead at the determination of the position point $P(x_p, y_p)$ face to a contour, and they are show through the relations (4).

It noted through $D = C \cup \text{Int}C$ the field determined by the close contour C and the rules are following:

$$R1: \text{If } S = \sum_{k=1}^r \theta_k = 360^\circ \vdash P(x_p, y_p) \in D \setminus C \text{ or } P(x_p, y_p) \text{ INTERIOR } C$$

$$R2: \text{If } S = \sum_{k=1}^r \theta_k = 0^\circ \vdash P(x_p, y_p) \notin D \text{ or } P(x_p, y_p) \text{ EXTERIOR } C \quad (4)$$

$$R3: \text{If } \neg R_1 \wedge R_2 \wedge \exists P(x_p, y_p) \equiv P_{h, h=1..n} \vdash P(x_p, y_p) \text{ BELONG TO VERTEX of } C$$

$$R4: \text{If } \neg R_1 \wedge \neg R_2 \wedge R_3 \vdash P(x_p, y_p) \text{ BELONG TO } C$$

In the first rule R1 (4) it decide that if the sum of angles θ_k is 360° the point $P(x_p, y_p)$ is interior of contour C .

In the second rule R2 (4) it decide that if the sum of angles θ_k is 0° the point $P(x_p, y_p)$ is exterior of contour C .

The three rule R3 (4) it decide that if the rules R1 and R2 are negatives and the point $P(x_p, y_p)$ is identic with a vertex of contour C , the point $P(x_p, y_p)$ is a vertex of contour C .

In the third rule R4 (4) it decide that if the rules R1, R2 and R3 are negatives, the point $P(x_p, y_p)$ belong to contour C .

COMPUTING ALGORITHM

The logical draft for determination the position relative of a point $P(x_p, y_p)$ face to a contour C it show in fig. 4.

It make the observation that in the logical draft it was considered for the first date the testing if the point $P(x_p, y_p)$ belong to a vertex of contour, than verification if the point P belong to contour through the relation $f_h(x_p, y_p) = 0$, where $h \in [1..m]$. If the test is negative, it call the subroutine SUM for computing the sum of angle θ_k for decide if the point is interior or exterior of contour. It imposed this sketch for the reduce the volume of computing.

So in the logical draft presented in fig.4, to ask the chain of primitives: $ch(V_h), h=1..m$, and the coordinates of point $P(x_p, y_p)$ of which position face to contour C ask to determined. The switch sw is fixed to 0. Then to call subroutine BELONG TO VERTEX where it tested if point $P(x_p, y_p)$ belong to a vertex of the contour C and the positive case it push $sw=1$ and apper the message "P(x_p, y_p) belong to vertex of contour", then the program it stoped. In the case sw it not equal with 1, to call subroutine BELONG TO CONTOUR C . In the case the point $P(x_p, y_p)$ belong to a primitive of chain $ch(V_h), h=1..m$, it push $sw=2$ and apper the message "P(x_p, y_p) belong to contour" then the program it stoped. In the case sw it differit of 2, it call the subroutine INTER and in this

subroutine first it tested if the primitives from chain $ch(V_h)$, $h=1..m$ are horizontal or vertical ; in the positive case it consider the extremities points of the primitive as points R_k, R_{k+1} . Then in the subroutine INTER computes the intersection points of the draws straight line $x=x_p$ and $y=y_p$ with contour C and results the points $R_{k,k=1..kk}$. Then call the subroutine SUM for compute the angles θ_k forms by the intersection points $R_{k,k=1..kk}$ and the point $P(x_p, y_p)$. If the sume S of the angles θ_k is 360° appear the message "P(x_p, y_p) interior contour" and programme is stoped. If the sum S of the angles θ_k is 0° , appear the message "P(x_p, y_p) exterior contour" and the programme is stoped. In the case sume S of the angles θ_k is not 360° or 0° it is an error.

CONCLUSION

Original algorithm proposed in this paper lead to obtain the correct results, resolves the difficult situations which apper if used the algorithm proposed by P.Winston [3] based of intersections number with contour. In fig. 5 are presented six possible positions of a point P face to six contours which are difficult of resolved by the algorithm proposed by P.Winston [3].

In fig.5a the point P is exterior to contour C and intersecting the contour C in the right once and the left of twice ,with once vertex at right and once vertex to left . In fig.5b the point P is interior to contour C and intersecting the contour C in the right once and the left twice ,with all three points vertex. In the fig.5c the pint P is exterior contour C and intersecting the contour C in the left into infinities points. In the fig. 5d the point P is interior to contour C and intersecting the contour C into infinities points in the left and the right. In the fig.5e the point P is interior the contour C and intersecting the contour C in the right twice and the left once ,with a vertex right and a vertex left. In the fig.5d the point P is interior of contour C and intersecting the contour C once in the left and twice in the right ,with a point of intersection is a point tangent with a primitive.

In the situations presented in fig.5a,5b,5c,5d ,the number of intersections to left or right of point P with the contours it is not conclusive.

The algorithm presented in this paper and proposed by Dora Florea is safe and resolve all the difficult situation very simple with the sum of angles . The algorithm was tasted with a program wrote in Visual Basic language and the results which obtained was very good.

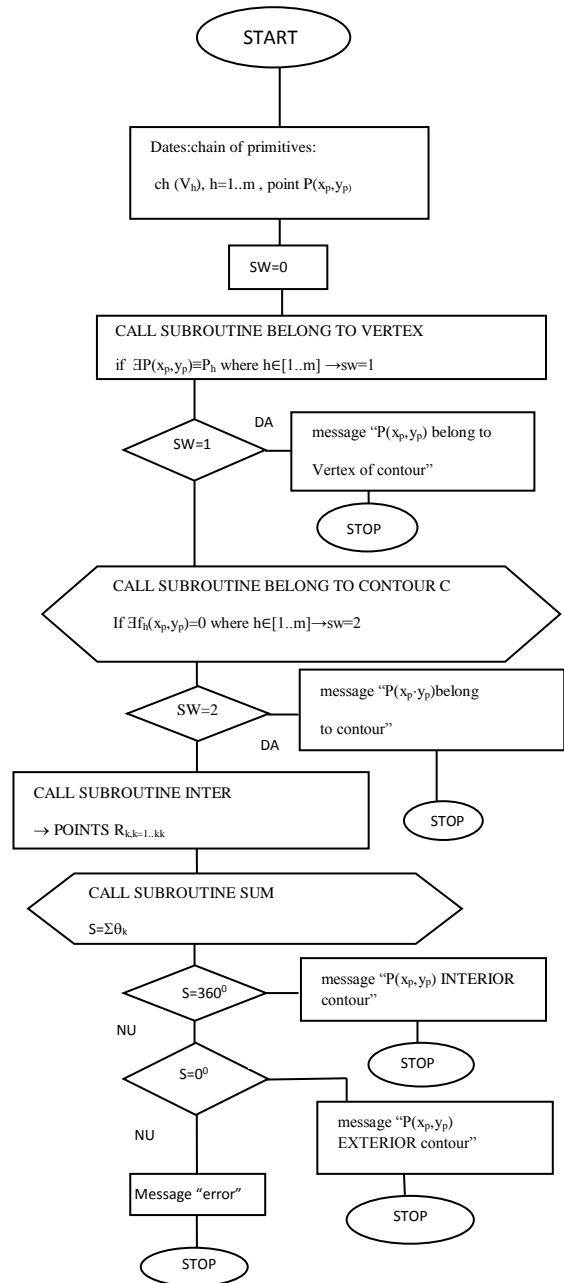


Fig.4 Logical draft for determination position relation of a point P(x_p,y_p) face to contour C

ACKNOWLEDGEMENTS

I take this opportunity to express my gratitude to the people who have been instrumental in the successful completion of this work.

AUTHOR BIBLIOGRAPHY

- [1] Patrick H. Winston, *Artificial intelligence*, Ed. Technique, Bucuresti , 1982.
- [2] Mihaela Malita, Mircea Malita, *Bazele inteligentei artificiale*, Ed. Tehnica, Bucuresti, 1987
- [3] Azriel Rosenfeld, *Digital Picture processing*, Academic Press, new York, 1976
- [4] Edwin M.E. and Downs F.L.Jr., *Geometrie*, Editura .Didactica si pedagogica , Bucuresti. 1983
- [5] H.G.Barrow, R.J.Poppkstone, *Relational Description in picture processing, Machine Intelligence* , Vol.6, University Press ,Scotland, 1981
- [6] Sergiu Corlat, *Algoritmi si probleme de geometrie computationala*, Ed. Prut International, 2009
- [7] N.Gricoiasiu, M.Iasinschi, A.Viciu, *Fise de Geometrie si Trigonometrie* ,Editura Dacia, Cluj-Napoca, 1978

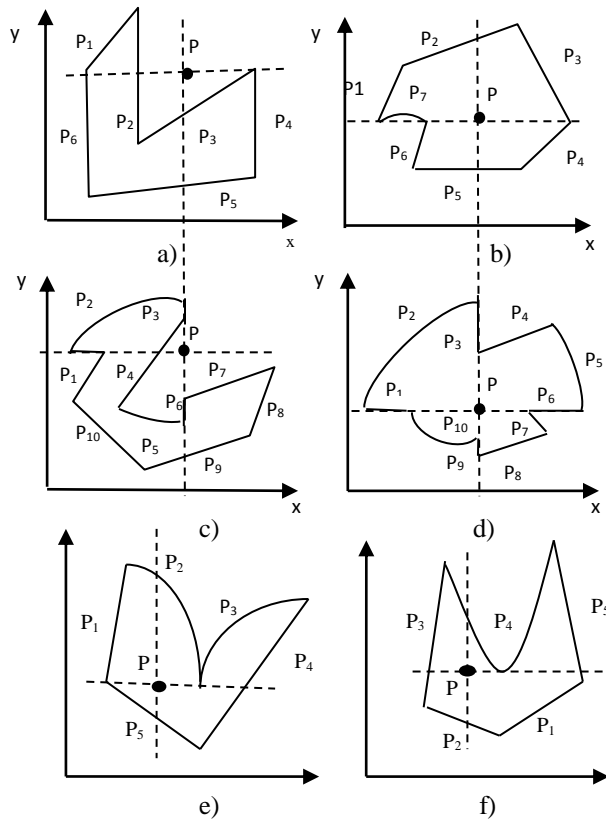
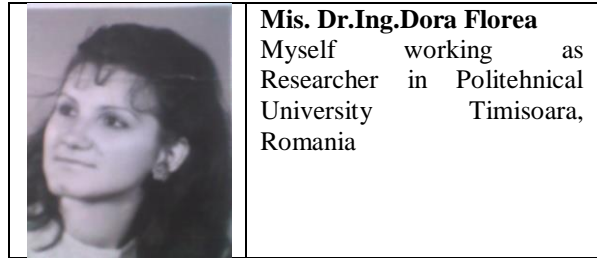


Fig.5 Some possible contours C and some possible position of point $P(x_p, y_p)$
a), b), c), d), e), f)